of all the collections would place the subject on a more satisfactory footing, and that ultimately the group, with all its modifications, may be thoroughly diagnosed.

EXPLANATION OF PLATE VI.

THE *Hamus* GROUP.

FIG. 1. *Alaria unicarinata*, sp.n. Dogger, Blue Wyke. York Museum. Front view.

,, 2, 2*a*. *Alaria unicarinata*, sp.n. Dogger, Blue Wyke. Bean Collection, British Museum. Back view and whorl enlarged.

,, 3, 3*a*. *Alaria Phillipsii*, d'Orb. Dogger, Blue Wyke. Leckenby Collection. Back view and whorl enlarged.

,, 4. *Alaria Phillipsii.* Millepore Rock, Cloughton. Leckenby Collection. Back view.

,, 5. *Alaria Phillipsii*, spinulose variety. Dogger (?), Peak. My Collection. Front view, enlarged twice.

,, 6, 6*a*. *Alaria pseudo-armata*, sp.n. Dogger, Blue Wyke. Leckenby Collection. Back and front view.

[The Hamus-group extends thus far only.]

,, 7, 7*a*. *Alaria bispinosa*, Phil. (variety). Kelloway Rock, Scarborough. York Museum. Back view, and spire enlarged.

,, 8, 8*a*. *Alaria bispinosa*, var. *elegans*. Cornbrash, Scarborough. Leckenby Collection. Back view and spire enlarged.

,, *9, 9*a*. *Alaria bispinosa*, var. *pinguis*. Kelloway Rock, Scarborough. Leckenby Collection. Back view and spire enlarged.

,, 10, 10*a*. *Alaria bispinosa*, var. *pinguis*. Kelloway Rock, Scarborough. Leckenby Collection. Back view and spire enlarged.

,, *11, 11*a*. *Alaria trifida*, Phil. Kelloway Rock, Scarborough. Leckenby Collection. Back view and whorl enlarged.

* The specimens thus marked occur on the same block of stone.

(*To be continued.*)

II.—GRAPHICAL METHODS IN FIELD-GEOLOGY.

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Introduction.

IN determining the actual position of strata from the appearances presented by their exposed edges in natural and artificial sections, certain mathematical problems are of constant occurrence. To a field-geologist who is not content with rough guesses founded on judgments by eye, the solution of these problems is a matter of importance, and methods have accordingly been given for some of those most frequently met with. These solutions take the form of (1.) trigonometrical formulæ, which can be applied only with the aid of trigonometrical and logarithmic tables; (2.) tables specially prepared from these formulæ for use in the field; (3.) graphical methods, requiring only a ruler, scale and protractor, which may be conveniently combined in one instrument. Of the first kind is the formula for deducing the true dip of strata from two apparent dips, given in Green's Geology (p. 341, 1st ed.), etc. Among special tables are those of Mr. Jukes for finding the apparent dip in any

direction from the true dip, and for connecting the dip, thickness and depth of beds: these tables are given in the appendix to the Survey Memoir on the Geology of the South Staffordshire Coal-field, and reproduced in Jukes's "Manual of Geology." Graphical methods have been used for finding the true dip from two apparent dips: a method partly graphical but requiring a table of cotangents is given in Phillips's "Treatise on Geology" (p. 298, 5th ed.), and also by the Rev. E. Hill (Geol. Mag. 1876, p. 334); a purely graphical method by Mr. W. H. Dalton (Geol. Mag. 1873, p. 332); and an approximate method by Mr. Penning (Geol. Mag. 1876, p. 236), reproduced in his "Field Geology." As Prof. Green has pointed out (*ib.* p. 377), the last-named method is equivalent to taking the angle for its tangent, and so applicable only to small angles of dip.[1] Mr. Dalton's solution (*loc. cit.* p. 334) of another question, to find the effect on strata already inclined of a second tilt in a new direction, is only an approximation, and cannot be applied if the dips are considerable. It is erroneously assumed that the inclination of the strata in a direction at right angles to that of the second tilt is unaltered by the tilting.

I propose to show that graphical methods are capable of wider and simpler application than they have yet received, and may be made really useful in field-work.

Various Modes of Treatment.

Questions relating to the intersection of planes, etc., may be treated in various ways, all equally simple. Firstly, we may draw figures to represent the planes themselves. For instance, let ABC, Fig. 1, represent the position of certain strata, ABD a horizontal plane, CD being vertical; then AB is the line of strike and AD, perpendicular to it, the direction of true dip; the angle CAD ($=X$ say) is the amount of dip and CBD ($=Y$) the apparent dip in a section making with the direction of true dip an angle ADB ($=Z$). Then we have directly (Fig. 1)

$$AD = CD \cot X, \quad BD = CD \cot Y, \quad AD = BD \cos Z.$$

$$\left.\begin{array}{ll} \text{Therefore} & \cot X = \cot Y \cos Z, \\ \text{or} & \tan Y = \tan X \cos Z, \end{array}\right\} \ldots\ldots (a)$$

which are the formulæ from which Mr. Jukes's tables are calculated.

Again, we may conveniently consider instead of the planes themselves the normals to them from a fixed origin O, and represent them by the points in which the normals meet a sphere of unit radius. For instance, in Fig. 2, let Z represent the horizontal plane, P the plane of the strata, Q that of the surface of the ground; then ZP represents the dip of the strata ($=X$), ZQ the slope of the ground ($=Y$), MN or PZQ the angle (Z) between the direction

[1] To indicate the degree of the approximation, suppose the two observed dips to make an angle of 60° with one another; then if the amounts of the dips be 15° and 20° respectively, the error in determining the direction of true dip by Mr. Penning's method is less than 1°; if the dips be 30° and 40°, it is about 4°; if 45° and 60°, it is 11°; and if 60° and 80°, the error amounts to 29°!

of dip and that of slope. Then PQ represents the angle (U) at which the surface of the ground cuts the strata, and we have at once

$$\cos U = \cos X \cos Y + \sin X \sin Y \cos Z \quad \ldots . (b).$$

This enables us to find the true thickness of a bed from the breadth of its outcrop, for the ratio of the former to the latter is evidently $\sin U$. Further, if the great circle PQ meet the horizontal great circle in R, R represents a vertical plane through the outcrop; so OR is perpendicular to the direction of outcrop, and since OM is perpendicular to the direction of strike, RM represents the deviation (V) of outcrop from strike. We readily obtain

$$\tan X \sin V = \cot PRM = \tan Y \sin (V - Z)$$

and so $$\tan V = \tan Y \sin Z \div (\tan Y \cos Z - \tan X) \quad . . . (c).$$

The problem of the "secondary tilt" is troublesome trigonometrically, though its graphical solution is sufficiently simple. I give only the results: if strata having an original dip X receive a secondary tilt of amount T in a direction making an angle S with that of the original dip, then the final dip Y, and the angle Z which it makes with the original dip are given by

$$\cos Y = \cos X \cos T - \sin X \sin T \cos S$$

and

$$\tan Z = \sin S \left(\cos X \cos \tfrac{1}{2} T - \sin X \sin \tfrac{1}{2} T \cos S\right)$$

$$\div \left\{\sin X \cos T + \cos X \sin T \cos S + 2 \sin X \sin^2 S \sin^2 \tfrac{1}{2} T\right\} \ldots (d).$$

For trigonometrical calculation the spherical projection is of course the most convenient, but as suggesting graphical constructions another projection presents advantages. The planes are represented by the points where they cut, not a sphere, but a horizontal plane at unit distance above the origin. Let the normals to the strata, the ground-surface and the horizontal plane, drawn through O, meet the plane of projection in P, Q and Z respectively; then OZ is unity, ZP is $\tan X$ and ZQ is $\tan Y$, X being the dip of the strata and Y the slope of the ground, and $PZQ = Z$, the angle between the directions of dip and slope (Fig. 3). In practice only the plane of projection with the traces on it of the various lines and planes is required (ZPQ in Fig. 4). In Fig. 3 $POQ = U$, the angle at which the strata are cut by the ground; if we imagine the triangle POQ turned about PQ into the plane of projection, we get Fig. 4, which at once leads to the construction given below (Problem vii.).

The constructions given for Problems x. xi. xii. and xiii. follow from equally simple considerations.

By this kind of projection we can represent the dip of any strata both in direction and amount by a line drawn from Z in a diagram on the plane of projection, for the line may be drawn to indicate by its direction the direction of the dip, and its length will be the tangent of the amount of dip. Similarly the slope of the ground can be represented by a line drawn from Z.

Use of the Protractor.

In accordance with the foregoing we require some convenient means of laying down at once on a diagram a length proportional to the tangent of any given angle. Such a means is furnished by a common protractor of the oblong form, graduated along a straight edge. This instrument serves not only as ruler, scale and protractor, but also as a rough table of tangents and cotangents. For the last purpose it is convenient to have it graduated with a second set of figures in addition to those usual on protractors, the second set increasing both ways from zero at the middle point Z of the straight edge. Take the breadth of the protractor OZ as unity; if the point P corresponds to say 35° reckoned from Z, then the angle ZOP is 35° and ZP is tan 35° (Fig. 5). It will be seen that for high angles (for angles greater than 60° in the figure) the application of this principle is less ready: for instance to lay down tan 70°, it is necessary to dot in the positions of O and Q and produce the lines ZP and OQ to meet in R, then ZR is the required length. The longer the protractor is in proportion to its breadth, the more degrees will be marked on the straight edge directly, but since the scale on which the tangents are represented depends on the breadth OZ, this should not be too small; say double the dimensions of Fig. 5. There would be some advantage in having O and Z not in the middle, but at one end of the protractor.

In the constructions which follow, a straight line will be said to *represent* the dip of any given strata when it is drawn from a fixed line Z —

(1) in the *direction* of the said dip, like the arrows on a common geological map, and

(2) of length corresponding to the *amount* of the dip, that is, the length given on the edge of the protractor from zero to the proper degree-mark.

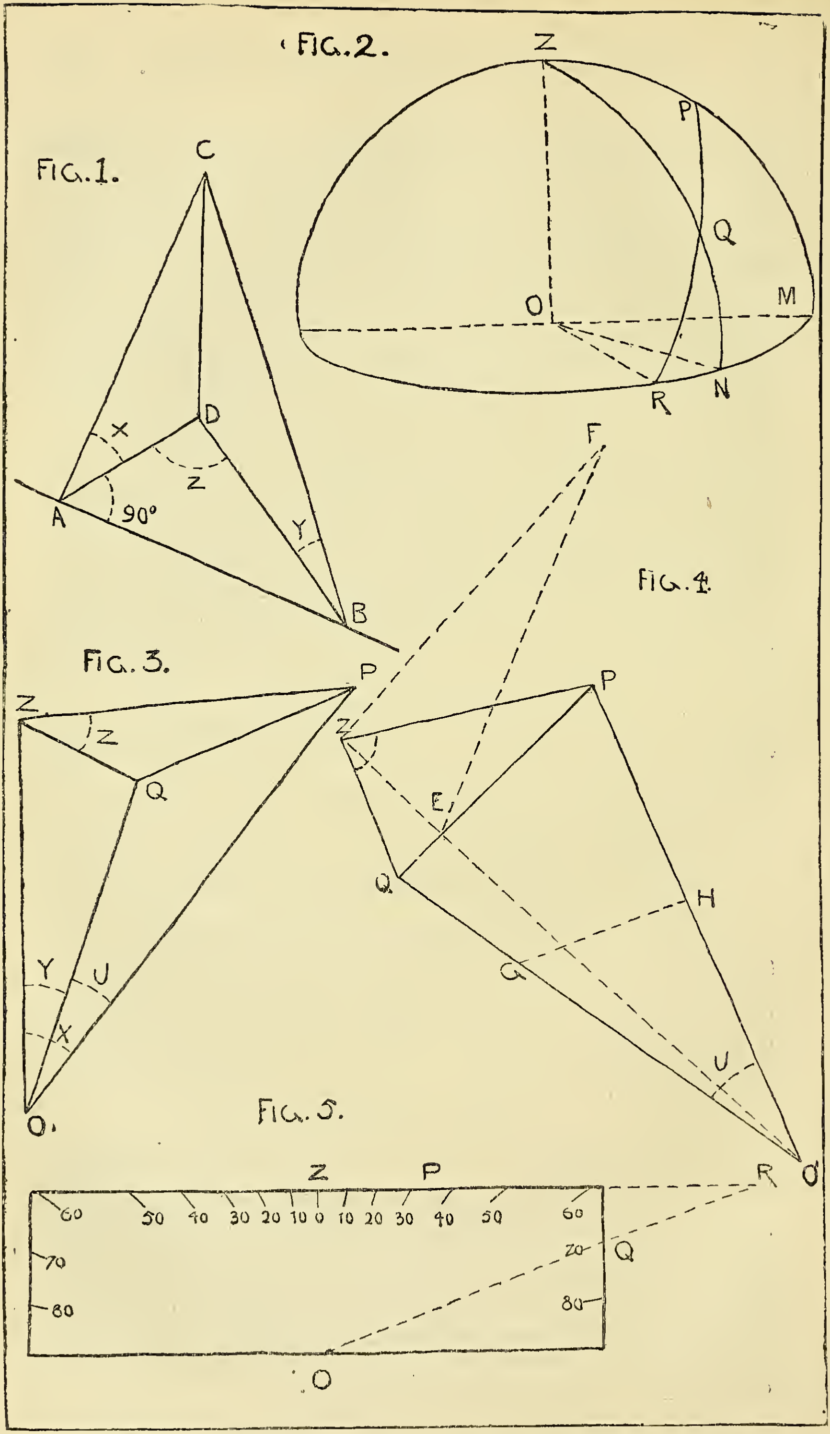
In this way the observations of the compass and clinometer are graphically recorded by one stroke, the protractor being used for the former purpose in the usual way, and for the latter in the manner described above. Similarly the slope of the ground or the inclination of any axis may be indicated, both in direction and amount, by a line on the diagram, the lines being always drawn from a fixed point of reference, Z.

Practical Applications.

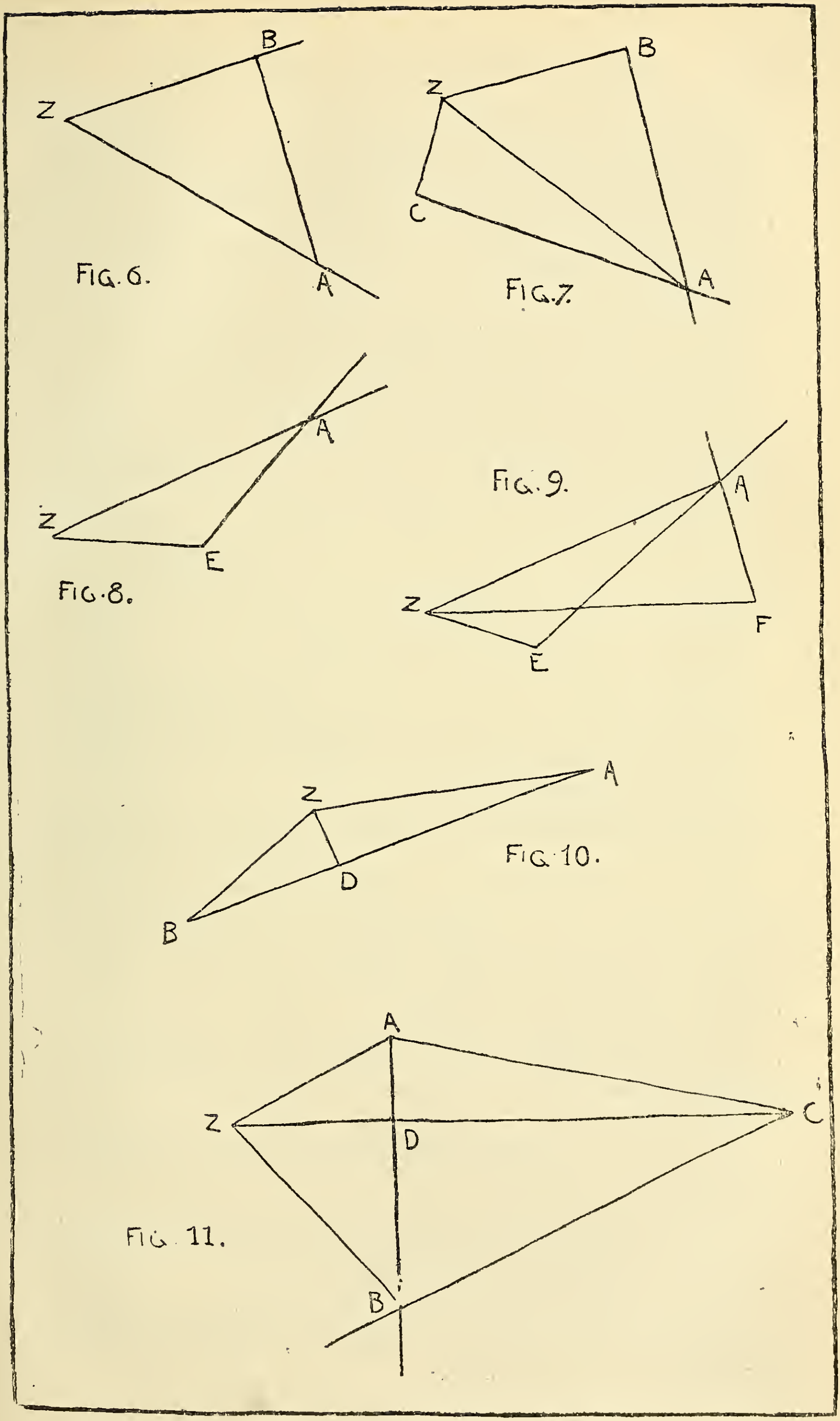
(i.) Given the direction and amount of full dip, to find the apparent dip in any given direction.

In Fig. 6 draw ZA to represent the full dip, ZB in the other given direction, AB perpendicular to it; then ZB, the part cut off, represents the apparent dip in magnitude as well as direction, and the amount of apparent dip may be read off by applying the edge of the protractor. The proofs of this and the two following constructions are evident from the second of the equations (a).

(ii.) Given the apparent dip in one direction, and the direction of full dip, to find the amount of the latter.



To illustrate Mr. Alfred Harker's paper on Graphical Methods in Field-Geology.



To illustrate Mr. Alfred Harker's paper on Graphical Methods in Field-Geology.

In Fig. 6 draw ZB to represent the apparent dip and ZA in the direction of full dip, the perpendicular BA will cut off a length ZA, which will represent the full dip.

(iii.) Given the apparent dip in two directions, to find the direction and amount of full dip.

In Fig. 7 draw ZB, ZC to represent the two observed dips, and perpendiculars to them at B and C, meeting in A; then ZA represents the full dip.

(iv.) Given the direction and amount of the dip of the strata and of the slope of the ground, to find the direction of outcrop.

In Fig. 8 draw ZA, ZE to represent the dip and the slope; then the direction of outcrop is perpendicular to AE.

To prove this and the two following constructions it is sufficient to notice that a horizontal line (OR in Fig. 2) drawn perpendicular to the direction of outcrop lies in the plane containing the normals OP and OQ. Equations (c) also give a proof.

(v.) Given the directions of strike and outcrop, to find the dip, the slope of the ground being known in direction and amount.

In Fig. 8 draw ZE to represent the slope, and from Z and E respectively draw lines perpendicular to the given strike and outcrop, meeting in A; then ZA represents the dip.

(vi.) Given the direction of outcrop in each of two localities where the slope of the ground is known in direction and amount, to find the direction and amount of the dip, supposed uniform (Fig. 9).

Draw ZE, ZF to represent the two slopes, and from E, F draw lines perpendicular to the respective directions of outcrop, meeting in A; then ZA represents the dip.

(vii.) Given the direction and amount of the dip and of the slope of the ground, to find the angle at which the strata are cut by the ground.

In Fig. 4 draw ZP and ZQ to represent the dip and the slope, ZE perpendicular to PQ, ZF perpendicular to ZE and equal to the breadth of the protractor; produce ZE until EO' is equal to EF, then $PO'Q$ is equal to the angle required.

(viii.) In the foregoing problem, given the breadth of the outcrop of any bed, to find its true thickness.

Measure off $O'G$ along $O'Q$ to represent on any convenient scale of magnitude the given breadth, then the length GH of the perpendicular drawn to $O'P$ will represent on the same scale the true thickness.

(ix.) Given in direction and amount the dips on the two sides of an inclined anticlinal axis, to find the direction and inclination of the axis.

In Fig. 10 draw ZA, ZB to represent the two dips, then the perpendicular ZD on AB represents the direction and inclination of the axis. A trigonometrical solution of this problem is given by Capt. F. W. Hutton, Geol. Mag. 1874, p. 44.

(x.) Strata having a dip given in direction and amount receive a secondary tilt given in direction and amount; to find the direction and amount of the resulting dip.

In Fig. 11 draw ZA to represent the original dip, and a line AB in the direction of the secondary tilt; draw ZD perpendicular to AB and produce until DC is equal to the breadth of the protractor; join AC and make an angle ACB equal to the amount of the secondary tilt; the lines AB and CB meet in B; then ZB represents the resultant dip.

(xi.) If in the foregoing problem the direction of the secondary tilt be at right angles to that of the original dip, the construction is much simplified.

Draw ZB (Fig. 6) to represent the original dip, and BA from B to represent the secondary tilt; then ZA represents the final dip.

When the angle the direction of the secondary tilt makes with that of the original dip does not differ widely from a right angle, and the amount of the tilt is small, the same construction gives an approximation to the true result; for instance, if the original dip be 30°, the amount of the tilt 20°, and the angle between their directions 60°, the error in the direction of the resulting dip is 3°, and in its amount 2°. For a strict solution the method in (x.) must be employed.

(xii.) Given the direction and amount of dip of strata which have suffered a tilt of known direction and amount, to find what the direction and amount of their dip was before tilting.

This is the same problem as (x.) worked backward, and a similar construction will suffice.

(xiii.) To find the direction and amount of the tilt required to change the dip of strata from a given initial direction and amount to a given final direction and amount.

In Fig. 11 draw ZA, ZB to represent the initial and final dips, draw ZD perpendicular to AB and produce until DC is equal to the breadth of the protractor; then AB is the direction of the required tilt and ACB its amount.

Further Remarks.

When an angle is near 90 degrees, its tangent is very great, and therefore the line representing it very long. Some of the above constructions are then slightly modified. For instance, if strata have a vertical position the line representing their dip in the manner described above would be of infinite length, but it is only necessary to draw this line for a short distance, and if another line has to be drawn to the infinitely distant extremity of the former, make it parallel to it, according to the geometrical principle that "parallel straight lines meet at infinity."

In some cases it is convenient to use a construction which employs not the angle of dip or slope, but its *complement*, that is, its defect from 90 degrees. The line indicating the dip is drawn in the direction of the dip, but of a length corresponding on the edge of the protractor to the complement of that angle. This line will be proportional to the cotangent of the angle of dip and will be longer or shorter according as the dip is small or great. As an example we will take the problem of finding the true dip from two observed

dips, and the method will be found to have some advantages over that given above (iii.).

Using Fig. 10, from Z draw ZA and ZB in the directions of the observed dips, but of lengths representing their complements; draw AB and ZD perpendicular to it; then ZD is the direction of full dip and corresponds in length to the complement of that dip. The edge of the protractor must be applied to ZD, and the reading thus obtained subtracted from 90 degrees to obtain the true dip itself. The proof of this is evident from the first of the equations (a). The method is equivalent to that of Mr. Hill cited above.

All the above constructions, with the exception of the second one in problem (xi.), are theoretically exact: the accuracy actually attained will of course depend upon the precision with which the drawing is performed. In some cases a small error in the drawing may give rise to a large error in the results, but this is due not to any fault in the method, but to the inadequacy of the data. For example, in problem (vi.), Fig. 9, if the slope of the ground in the two localities be nearly the same, and the directions of outcrop nearly the same, the lines EA and FA will meet at a small angle, and any error in drawing them will produce a magnified error in the position of A, and therefore in the direction and magnitude of ZA; but this is because the observations are insufficient to determine the dip of the beds with any degree of accuracy.

The foregoing examples are enough to illustrate the wide applications of graphical methods; solutions of other problems on similar lines will suggest themselves.

III.—Notes on the Appendages of Trilobites.

Note to accompany Three Woodcuts of *Asaphus megistos*, a Trilobite discovered by Mr. James Pugh, near Oxford, Ohio, in the upper portion of the Hudson River Group.

We are indebted for the Woodcuts accompanying this note to the courtesy of Dr. John Mickleborough, whose paper on the Locomotory Appendages of Trilobites we published in our February Number, p. 80.

They serve admirably to confirm the observations of Mr. E. Billings (published in the Geological Magazine for 1871, Vol. VIII. Pl. VIII. pp. 289–294) on the appendages of *Asaphus platycephalus*. In Fig. 1, *a, a*, mark the position of the anterior pair of appendages; *b, b*, the 10th pair; *c*, the articulation between the carpus and propodos; *d*, the articulation between the propodos and dactylus; *e*, the lines to the letter *e* mark the position of the lamelliform ? branchiæ beneath the pygidium.

Fig. 2 represents the upper surface of specimen reduced to nearly one-third natural size.

Fig. 3 shows the mould into which Fig. 1 fits. *a, a*, mark the moulds of the bases of the anterior pair of appendages. *b, b*, of the 10th pair of appendages; the lines leading to *c* inclose the probable space to which the lamelliform ? branchiæ were attached; *d* marks the position of the left maxillipede; *e* the left angle of the hypostome.